

Goat problem equation

Let x be the length of the rope required to tether a goat to the edge of a circular field with radius one so that the goat can graze exactly half the area of the field. Then

$$2 \sin^{-1}(x/2) + x^2 \cos^{-1}(x/2) - x\sqrt{1 - x^2/4} = \pi/2$$

Solving this (with Newton's method) gives $x = 1.158728473$ (to 10 s.f.).

Bird problem equation

Let x be the length of the string required to tether a bird to the side of a spherical cage with radius one so that the bird has access to exactly half the volume of the cage. Then

$$8x^3 - 3x^4 = 8$$

Solving this (with Newton's method) gives $x = 1.228544864$ (to 10 s.f.).

In fact

$$x = \frac{2}{3} + \frac{1}{2} \left(\sqrt{a} - \sqrt{\frac{16}{3} - a + \frac{128}{27\sqrt{a}}} \right)$$

where

$$a = \frac{16}{9} + \frac{4}{3} \left((4 + \sqrt{8})^{\frac{1}{3}} + (4 - \sqrt{8})^{\frac{1}{3}} \right)$$

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