A very silly way to calculate pi

$$\frac{\sqrt{\pi}}{2} = \lim_{x\to\infty}\left(\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)n!}\right)$$

Proof:

This is just the probability integral $\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$ in disguise. At http://www.maths.lancs.ac.uk/~jameson/timint.pdf is my brother Tim's proof of this integral by volume of revolution (which he came up with at the age of just 16).

Recall that

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Replace x with $-x^2$ in this to get

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots$$

Integrating termwise gives us

$$\int_{-X}^{X} e^{-x^2}dx = \left[x - \frac{x^3}{3} + \frac{x^5}{5*2!} - \frac{x^7}{7*3!} + \cdots\right]_{-X}^{X}$$

$$= 2\left(X - \frac{X^3}{3} + \frac{X^5}{5*2!} - \frac{X^7}{7*3!} + \cdots\right)$$

i.e.

$$\frac{1}{2}\int_{-X}^{X} e^{-x^2}dx = \sum_{n=0}^{\infty}(-1)^n\frac{X^{2n+1}}{(2n+1)n!}$$

Let $X \to \infty$ and we are done.

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