Weird integrals

$$\pi\cot(\pi x) = \frac{1}{x} - 2\int_0^\infty \frac{\sinh(xt)}{e^t - 1}\,dt \quad \text{for } |x| < 1$$

$$\pi^2\csc^2(\pi x) = \frac{1}{x^2} + 2\int_0^\infty \frac{t\cosh(xt)}{e^t - 1}\,dt \quad \text{for } |x| < 1$$

Obviously the second one is the result of differentiating and negating the first.

Tested both numerically. Both work well as long as $|x|$ isn't too close to 1.